ABSTRACT
The aim of this study is estimation of social welfare in Russian regions in 2004-2014 based on the A. Sen abbreviated function, the assessment of regions convergence in welfare and factors contributing to it. We adjusted the A. Sen welfare function by the cost of living in regions and presented it as a four-factor multiplicative model.
By use of the coefficient of variation (CV) we estimated the inter-regional inequality in welfare in statics and dynamics. The technique of decomposition of the squared CV for logarithm of welfare function enabled us to evaluate the contribution of main and intersect factors to Russian regions’ convergence in welfare. As a result we assessed Russian regions social welfare in dynamics and evaluated factors contributing to its growth. Based on the weighted CV we discovered the negative impact of the recession of 2009 on the regions convergence in welfare, and unweighted CV even more revealed the turning point in the convergence tendency occurring in 2012.
We discovered that in statics the redistributive factor makes the greatest and increasing contribution to inter-regional convergence in welfare, whilst the cost of living and intra-regional income inequality factors have moderate and decreasing influence on it.
The dynamic analysis revealed two factors predominantly contributing to regions’ temporal convergence in welfare, namely growing convergence in GRP per capita and significant but unstable influence of distributive factor. Two other factors, the cost of living and intra-regional income inequality, counteracted to temporal convergence mainly due to attenuation of their negative correlation to nominal and real income per capita respectively. The results obtained may be applicable to both inter-budgetary and regional policy development.
Keywords: Abbreviated functions, Convergence, Decomposition, Factors, Inequality, Region, Coefficient of Variation, Welfare

1. INTRODUCTION
The inter-regional inequality in welfare is a phenomenon naturally occurring in countries with considerable spatial diversity in the resources provision and in the level of regional economic development. Such an inequality may increase because of migration of resources and may
decrease due to redistributive policy and management of institutional environment. Many authors study inequality in gross regional product (GRP) or income per capita (Lakner, Milanovic, 2013). Apparently, these indicators are incomplete and do not fully describe the well-being. The welfare in a broad sense includes a number of elements beyond and behind GDP ensuring people satisfaction with life.

The simple welfare functions are based on the main development indicator and its adjustment by other indicators characterizing the socio-economic processes accompanying the economic development and affecting the people well-being. The economists who initially studied the problem usually considered two main side effects of development commonly appearing at its lower stages: environmental (pollution) and social (increase in inequality) costs.

Later researchers proposed complex assessment of welfare based on various parameters of development, such as consumption, leisure time, life expectancy, level of health and education, pollution etc. They devised integrated indices of welfare, e.g. the Human Development Index and the Quality of Life Index, now frequently used for comparison of states or regions and for measurement of spatial inequality (Grasso, Canova, 2008; Jordá, Sarabia, 2015). These indices are applied for assessment of welfare of Russian regions as well (Rating of Russian regions, 2015; Human Development Report, 2013). Nevertheless, the more complex measurement of welfare was presented by H. Daly and J. Cobb (Daly, Cobb, 1989) who developed the Index of Sustainable Economic Welfare (ISEW). Following some modifications it was transformed into the Genuine Progress Indicator (GNI). Today these measures are also employed in a range of comparative studies (Bleys, 2013; Andrade, Garcia, 2015).

However, the calculation of the integral index of welfare requires collection and processing of a large amount of information, and some data may be obtained only through sociological surveys. Accordingly to (Lawn, 2003), the methods of construction of integrated indices are still controversial. Whilst (Jordá, Trueba, Sarabia, 2013) indicate that these methods should take into account that welfare components may be complementary or substitutionary. Therefore, we postpone the complex assessment of the welfare of Russian regions until appropriate time.

In the current research we focus on abbreviated welfare functions taking into account only level of development and income inequality because they are evidently interrelated. According to previous researches, the vast majority of Russian regions demonstrate growing inequality over the last decade and movement on the ascending branch of the S. Kuznets curve (Malkina, 2014a). It explains our primary interest to adjustment of the regions average income with income inequality that would obviously lower the estimations of regions’ convergence.

At first the inequality-based approach to welfare was suggested by A. Sen in the form of “the abbreviated social welfare functions” (Sen, 1976). The A. Sen welfare function takes the following form: $S = \mu \cdot (1 - G)$, where $\mu$ – average income per capita, $G$ – the Gini coefficient.

Alternatively, N. Kakwani (Kakwani, 1981) incorporated a penalty for inequality in the welfare function and additionally took into account the reduction in life expectancy. R. Lambert represented the welfare function as direct dependency on income and inverse dependency on inequality (Lambert, 2002). Some advanced welfare functions incorporate people’s attitude to inequality. Thus, in the C. Dagum’ approach an individual’s welfare depends on the number of people with higher earnings (Dagum, 1990). In other approaches the Gini coefficient was replaced by the A.B. Atkinson index, based on the utilitarian function of income.
with the diminishing marginal utility and taking into account the society perception of inequality (Atkinson, 1970). In this function the “e” parameter indicates the level of public aversion to inequality (Carlsson, Daruvala, Johansson-Stenman, 2005; Howarth, Kennedy, 2016). Some authors used all the family of “inequality-adjusted aggregate welfare functions” to complete the picture (Gruen, Klasen, 2008).

In our paper, we employ the A. Sen abbreviated function for assessment of the Russian Federation regions social welfare. Bearing in mind different level of prices and the cost of living in Russian regions, we substitute the nominal income for the real income. Further, based on the calculation of indices of inter-regional inequality we evaluate the degree of convergence / divergence of Russian regions in welfare in 2004-2014 years. Then we propose the method of decomposition of inequality index, which allows us to assess the factors’ contributions to the inter-regional convergence (divergence) in the A. Sen welfare.

2. DATA AND METHODS

Our research is based on the official sources of information provided by the Federal Service of State Statistics of RF. We used annual data on 80 Russian regions for 2004-2014, including the number of population, GRP, personal incomes, the cost of a fixed basket of consumer goods and services, incomes inequality (measured by the Gini coefficient).

The algorithm of our study includes several stages.

2.1. Measurement of regions welfare

We presented the A. Sen social welfare function as the product of the following factors:

\[ S_i = \frac{Y_i}{N_i} \times \frac{D_i}{Y_i} \times \frac{CI_i}{CI} \times (1 - G_i) = \frac{y_i \cdot \hat{c}_i \cdot c_i \cdot g_i}{n_i}, \]  

where \( y_i = \frac{Y_i}{N_i} \) — nominal gross regional product (GRP) per capita in “i” region; \( \hat{c}_i = \frac{D_i}{Y_i} \) — the ratio of personal incomes to GRP in each region; \( c_i = \frac{CI_i}{CI} \) — index, inverse to the relative cost of living in “i” region, calculated as the ratio of the cost of a fixed basket of consumer goods and services in the country to the cost of this set in each particular region; \( g_i = 1 - G_i \) — income erosion, \( G_i \) — intra-regional Gini coefficient for the nominal personal incomes in “i” region. Note that due to the lack of statistical data on the cost of the consumer basket for the various groups of the population it is impossible to calculate the Gini coefficient for the real incomes. At each subsequent stage we obtained new parameter of the model: \( n_i = y_i \cdot \hat{c}_i \) — nominal personal incomes per capita; \( r_i = y_i \cdot \hat{c}_i \cdot c_i \) — real personal incomes per capita in “i” region.

Further we calculated the logarithms for the A. Sen welfare for three reasons. Firstly, by this way we pass to the welfare utility function with diminishing marginal utility. Secondly, thus
we approach normal distribution of welfare. Thirdly, the indices based on logarithms of some variable are easily decomposed, when this variable is represented in multiplicative form.

\[
\ln(S_i) = \ln(y_i) + \ln(\partial_i) + \ln(c_i) + \ln(g_i) .
\]  

(2)

Next, to simplify we make the following substitutions: \( S^*_i = \ln(S_i) \); \( y^*_i = \ln(y_i) \); \( \partial^*_i = \ln(\partial_i) \); \( c^*_i = \ln(c_i) \); \( g^*_i = \ln(g_i) \).

2.2. Evaluation of inter-regional inequality in welfare

The inter-regional inequality is usually assessed with different methods (Ayala, Jurado, Pedraja, 2010, p. 240). In our study for this purpose we chose the coefficient of variation (CV) both for the logarithms of the A. Sen welfare and its components. The population-weighted CV for each “x” variable measures the scale of inequality, while the unweighted CV measures the sharpness of inequality. The first one takes the form:

\[
CV_x = \sqrt{\frac{\sum_{i=1}^{n} \rho_i \cdot (x^*_i - x^*)^2}{x^*}}.
\]  

(3)

Here \( x^*_i \) – average value of some variable in “i” region, \( \rho_i \) – the share of “i” region in total population of the country, \( x^* = \sum_{i=1}^{n} \rho_i x^*_i \) – the mean value of \( x^*_i \) variable, \( n \) – the number of regions. To obtain the unweighted CV we should replace the shares of regions in population \( \rho_i \) for their shares in number of regions, 1/n.

2.3. Decomposition of inter-regional inequality in welfare

We propose following method of decomposition of the squared CV for the logarithm of the A. Sen welfare function:

\[
CV^2[S^*] = \left( \frac{y^*}{S^*} \right)^2 \cdot CV^2[y^*] + \left( \frac{\partial^*}{S^*} \right)^2 \cdot CV^2[\partial^*] + \frac{2 \cdot Cov[y^*,\partial^*]}{S^*} + \left( \frac{c^*}{S^*} \right)^2 \cdot CV^2[c^*] + \frac{2 \cdot Cov[p^*,c^*]}{S^*} + \left( \frac{g^*}{S^*} \right)^2 \cdot CV^2[g^*] + \frac{2 \cdot Cov[r^*,g^*]}{S^*}.
\]  

(4)

We denoted \( \left( \frac{x^*}{S^*} \right)^2 \cdot CV^2[x^*] \) as \( M(x) \) for all main \( x^* \) variables. Accordingly we made substitutions \( \frac{2 \cdot Cov[z^*,x^*]}{S^*} = \text{Inter}(z,x) \) for all intersections of each main \( x^* \) variable with the collected parameter \( z^* \) achieved at each previous stage of the A. Sen function formation. Consequently, full decomposition of the squared CV takes the form:

\[
CV^2[S^*] = M(y) + M(\partial) + \text{Inter}(y,\partial) + M(c) + \text{Inter}(n,c) + M(g) + \text{Inter}(r, g).
\]  

(5)

Thus, inter-regional inequality in budget expenditures per capita is fully decomposed to the contribution of main parameters and intersections of the model (1).
Based on our previous research (Malkina, 2014b), we can assume the positive correlation between the Gini coefficient and average real personal income in Russian regions. Besides, in the regions with higher nominal personal incomes a higher level of consumer prices should be observed on average. Finally, regions with a higher GDP per capita tend to have lower share of personal incomes in GRP. Given our parameters designed, all the Inter values should be negative.

Now we can formulate the hypothesis of the study. We presume that in the chain “nominal GRP per capita → nominal personal incomes per capita → real personal incomes per capita → the A. Sen welfare per capita” each subsequent parameter should demonstrate even less inter-regional inequality compared to the previous one.

Ultimately, the decomposition of the squared CV for the A. Sen welfare function by components enables us to evaluate the contribution of main and intersect parameters of the model (1) into the Russian regions’ convergence (divergence) in welfare both in statics and dynamics.

3. RESULTS AND DISCUSSION

3.1. Analysis of the A. Sen welfare functions in Russian regions

By calculation of social welfare using formula (1) we have obtained the estimations and ranks of Russian regions by welfare in dynamics. The figure 1 demonstrates the distribution of welfare among Russian regions in 2014. According to the A. Sen function, the most affluent Russian regions are neighboring mining territories with gas and oil production, namely: Yamalo-Nenets (S=26245) and Khanty-Mansi Autonomous Okrug – Yugra (S=19622), located in the Ural Federal District, and Nenets Autonomous Okrug (S=26405), subject of the Northwestern Federal District. They are followed by the Moscow city (S=21257), Russian managerial and financial center, and some extra mining territories located in the Far East: Chukotka Autonomous District (S=20640) and Sakhalin Oblast (S=19193), and Republic of Tatarstan (19184), located in the Volga Federal District. The least prosperous regions, according to the A. Sen function, are also some boarder territories: Republic of Kalmykia in the Southern Federal District (S=8395), Republic of Tyva (S=9405) and neighboring Republic of Altai (9858), located in the Siberian Federal District.

The group of deprived regions also includes three backward North Caucasian republics: Karachay-Cherkess Republic (S=10481), Kabardino-Balkar Republic (S=11084) and the Republic of Ingushetia (S=10111).
The average population-weighted welfare in Russian regions has increased 4.48 times over 2004-2014. At the same time the largest increase in the welfare is observed mainly in the poorer regions: Chechen Republic (8.47 times), Republic of Ingushetia (7.75 times), Republic of Dagestan (6.56), Ivanovo Oblast (6.61), and the Republic of Adygea (6.43). The first three are again the subjects of the North Caucasian Federal District. The lowest growth of welfare is marked both in some rich regions: Tyumen region, including Khanty-Mansi and Yamalo-Nenets Autonomous Okrugs (3.53 times), the city of Moscow (3.73) and St. Petersburg (3.69), and in some relatively poorer regions: Republic of Karelia (3.59), Kemerovo Oblast (3.59), Republic of Komi (3.62) and the Tomsk Oblast (3.69). The last four regions have lost in the welfare ranking 38, 26, 11 and 39 points respectively. Besides, the large drop in welfare ranking is observed in Pskov Oblast (39 p.), Volgograd Oblast (32 p.) and Leningrad Oblast (26 p.).

By means of logarithmic method of factor analysis we have decomposed welfare growth in Russian regions and measured the main components' contribution. Some results are presented in the table 1. It indicates that the welfare increase is predominantly attributable to the growth of GRP, whereas two other components, the share of personal income and income inequality, have demonstrated a moderate influence on welfare growth. Meanwhile, the factors contributions to the welfare growth differ a lot in the regions. Thus, the GRP factor showed a greatest impact on welfare of some North Caucasian republics:
Chechnya, Ingushetia and Dagestan, and again of the Republic of Adygea, the subject of the Southern Federal District. These are all backward regions, in fact demonstrating catching-up effect in development. But simultaneously large increase in GRP per capita has been observed in the Sakhalin Oblast, one of the highest-level welfare regions. On the contrary, the lagging growth of GRP per capita in Vologda, Pskov, Kemerovo, Tomsk, Omsk and Tyumen regions has considerably let them down in the welfare ranking.

Table 1: The results of decomposition of the welfare growth rate, % (author calculations)

<table>
<thead>
<tr>
<th></th>
<th>Minimum value (region)</th>
<th>Maximum value (region)</th>
<th>Average (Russian Federation value)</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>GRP</td>
<td>196,9 (Vologda Oblast)</td>
<td>700,5 (Chechen Republic)</td>
<td>331,9</td>
<td>85,8</td>
</tr>
<tr>
<td>Share of personal incomes in GRP</td>
<td>-173,7 (Sakhalin Oblast)</td>
<td>179,1 (Lipetsk Oblast)</td>
<td>8,6</td>
<td>51,6</td>
</tr>
<tr>
<td>Relative cost of living</td>
<td>-35,3 (Vladimir Oblast)</td>
<td>68,0 (Chukotka Okrug)</td>
<td>0,0</td>
<td>18,4</td>
</tr>
<tr>
<td>Income inequality</td>
<td>-45,4 (Chechen Republic)</td>
<td>54,2 (City of Moscow)</td>
<td>7,3</td>
<td>11,5</td>
</tr>
<tr>
<td>Welfare (based on the A. Sen function)</td>
<td>253,0 (Tyumen region)</td>
<td>746,9 (Chechen Republic)</td>
<td>347,8</td>
<td>87,1</td>
</tr>
</tbody>
</table>

The advancing increase in the share of personal income in GRP has picked Ivanovo and Lipetsk Oblast up in the welfare ranking. However, the negative contribution of this factor to welfare of the Moscow city (-80.2%) has not influenced its rank, albeit has decreased its gap with less prosperous regions.

In some regions the relative cheapening of the consumer basket compared to the average state-wide one has led to additional gain in welfare. Apart from Chukotka mentioned in the table 1, it is the case of some other Eastern territories such as Sakhalin Oblast and the Republic of Sakha (Yakutia), and also of the Republic of Ingushetia. Simultaneously, some Western and Central regions, specifically Smolensk, Pskov and Vladimir Oblast, have demonstrated significant negative impact of the relative growth in cost of living on change in the welfare. The similar negative influence is established in some Northern Caucasian regions: Karachay-Cherkess and Chechen republics.

Ultimately, considerable decrease of intra-regional inequality in the Moscow city proved to be an important factor maintained its welfare gap with other regions as well as its high position in the ranking. The backward Northern Caucasian republics, although growing at a higher rate, have been again thrown back in welfare due to rise in intra-regional inequality accompanying their development.

3.2. Analysis of the Russian regions convergence / divergence in the A. Sen welfare

Both population-weighted and unweighted regional disparities in welfare, assessed by the formula (3), point at the regions convergence (Fig. 2). In general, the inter-regional inequality in welfare has decreased approximately by 43% over 11 years.

However, the unweighted approach demonstrates a smooth process of regions’ convergence in welfare with decreasing rate after 2008, turning to inverse tendency of divergence since 2012. In comparison, the weighted approach demonstrates irregular tendency of convergence.
during the entire period analyzed, but it proves to be more sensitive to the 2009 recession. The distinct results obtained through the two approaches are obviously related to the concentration of population in the more prosperous regions, as well as different impact of crises on poor and rich regions.

![Figure 2: Tendencies of convergence of Russian regions by the A. Sen welfare function](image)

*Figure 2: Tendencies of convergence of Russian regions by the A. Sen welfare function (author calculations based on the Russian Federation Federal State Statistics Service data)*

### 3.3. The results of decomposition of the Russian regions inequality in welfare

The decomposition of the squared CV based on the formulae (4) and (5) demonstrates positive impact of the main four factors and negative impact of three intersect factors on the total inequality for each year (Figure 3).

![Figure 3: The results of decomposition of the inter-regional inequality in welfare in Russia](image)

*Figure 3: The results of decomposition of the inter-regional inequality in welfare in Russia (author calculations based on the Russian Federation Federal State Statistics Service data)*

Initially, we consider the group of the main factors of inequality as independent. The inter-regional differences in welfare are largely attributable to uneven distribution of GRP relatively to distribution of population in the regions. The share of this factor among all main factors of the model has been predominant throughout all the period under study, but gradually decreasing (76.9–71.0%). The inter-regional unevenness of another factor, the share of
personal incomes in GRP, has increased, thus its relative influence has grown, but even in 2014 it did not exceed 24%. On the contrary, the contributions of the cost of living and intra-regional inequality to welfare disparities were smallest and only decreasing over time. However, it should be noted that the main factors were acting along with intersect factors, all of which negatively contributed to regions’ welfare inequality, thereby supporting their convergence in statics.

Firstly, in the regions with higher level of GRP per capita the share of personal incomes in GRP proved to be lower on average. This factor, \textit{(re)distributive by nature}, had the greatest impact on convergence. Moreover, its contribution has increased 1.81 times over 11 years, supporting equalization of the welfare at the stage of income distribution.

Secondly, in the regions with higher level of nominal income per capita the average level of consumer prices turned to be higher. However, due to contraction of potential convergence in cost of living, the impact of inflationary factor has decreased almost twice (by 48%).

Thirdly, in the regions with higher level of real income per capita the intra-regional income disparities are again higher on average. Due to the Sigma-convergence of poor and rich regions, the contribution of the “income erosion” factor to the welfare leveling has fallen 3.8 times.

The proportional method of factor analysis allowed us to calculate the factors’ contributions to the regions welfare convergence in dynamics. In the table 2 we have combined the influences of each main component of our model (formula 1) and its intersection with the parameter achieved at previous stage of the welfare formation.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
& Contribution to the convergence & & & \\
& GRP production & Distribution & Inflation & Intra-regional inequality & Total \\
\hline
2007/2004 & -37.71 & -77.66 & 54.09 & 29.28 & -32.00 \\
2009/2004 & -146.95 & -38.04 & 82.53 & 50.41 & -52.04 \\
2010/2004 & -142.90 & -63.23 & 92.13 & 57.36 & -56.64 \\
2011/2004 & -144.29 & -70.99 & 95.15 & 59.70 & -60.44 \\
2012/2004 & -175.26 & -56.81 & 102.59 & 66.25 & -63.23 \\
2014/2004 & -211.37 & -43.63 & 114.77 & 72.73 & -67.50 \\
\hline
\end{tabular}
\caption{The factors’ contribution to Russian regions convergence in the A. Sen social welfare, by an accrual basis since 2004, \% (author calculations)}
\end{table}

Based on the results obtained, we may affirm the positive increasing influence of GRP convergence as well as positive unstable influence of incomes (re)distribution on the reduction in regions’ welfare disparities. On the contrary, two other factors, the cost of living and the intra-regional inequality, have been evidently counteracting to the process of convergence in dynamics.
4. CONCLUSION

The previous researchers have developed a number of approaches to assessing the welfare of countries and regions, some of which were referred to the abbreviated social welfare functions based on income per capita and income inequality. The adoption of the A. Sen welfare function allowed us to estimate the welfare of Russian regions in dynamics for 2004-2014. Based on the measurement of inter-regional inequality in welfare we have revealed the tendency of the Russian regions convergence, which was temporarily disrupted with the shock of divergence in 2009 (according to the weighted coefficient of variation for logarithm of the A. Sen function) and even turned to the opposite tendency in 2012 (according to the unweighted coefficient of variation for this function).

We suggested the four-factor multiplicative model for the A. Sen welfare function and proposed the logarithmic technique for its decomposition. This allowed us to evaluate impact of the main components and the intersections on the regions’ convergence in welfare. The growing influence of production factor along with significant yet unstable impact of redistribution factor indirectly indicates the efficiency of inter-budgetary equalization policy. It also confirms the findings of one previous econometric study (Yushkov, 2015). The positive correlations between regions’ average income, on one side, and cost of living and internal inequality, on the other side, have contributed to some reduction in regional disparities in statics. This result is consistent with (Malkina, 2014a). However, the faster growth of the cost of living and inequality in the poorest regions restrained the regions convergence in welfare in dynamics.

The further extension of the research is possible by way of adaptation of complex welfare functions, e.g Index of Sustainable Economic Welfare and Genuine Progress Indicator for the Russian regions, with paying attention to embedded features of the Russian statistics. The development of such measures should facilitate a more accurate assessment of convergence / divergence of the Russian regions in terms of welfare, taking into account diverse social, environmental, demographic and other costs and benefits of development.

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LITERATURE: